A General Four-Fermion Effective Lagrangian for Dirac and Majorana Neutrino-Charged Matter Interactions

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Abstract

Given the most general Lorentz invariant four-fermion effective interaction possible for two neutrinos and two charged fermions, whether quarks or leptons, all possible $2 \rightarrow 2$ processes involving two neutrinos, whether Dirac or Majorana ones, and two charged fermions are considered. Explicit and convenient expressions are given for the associated differential cross-sections. Such a parametrization should help assess the sensitivity to physics beyond the Standard Model of neutrino beam experiments which are in the design stage at neutrino factories.

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1 Introduction

Neutrino physics is called to play an important role in fundamental physics, certainly for the decades to come, ranging from high energy particle physics to astroparticle physics and cosmology. Neutrinos hold the key to some of the present-day mysteries in these fields, with the potential for profound breakthroughs in our search of the unification of all interactions. However in this endeavour, the lead is certainly to come from experiments: they must identify which, if any, of the many theoretical avenues that have been imagined, has actually been chosen by Nature. Thus ambitious projects are in the making, ranging from neutrino telescopes to accelerator experiments and neutrino factories.

Given this context, it should be of interest to have available a general model independent parametrization of large classes of processes within an effective description relevant for a given energy range. For example in the 1950's, such a general four-fermion effective interaction[1] has enabled the identification of the $(V-A)$ structure of the weak interaction in β-decay and is still used in modern precision measurements.[2] A similar parametrization is also of relevance to precision studies in the muon sector looking for physics beyond the Standard Model within purely leptonic processes.[3, 4] Likewise, such a parametrization of semi-leptonic muonic interactions is available in the intermediate energy range of nuclear muon capture.[5] Given the eventual advent of neutrino factories, an analogous general parametrization should thus be of interest, in order to assess the potential of any given experiment based on neutrino beams to constrain the parameter space of possible physics beyond the Standard Model (SM) within the neutrino sector.

In this note, we wish to report briefly on such an analysis and some of its results,[6, 7] based on the most general four-fermion effective interaction possible of two neutrinos and two charged fermions (whether leptons or quarks) of fixed "flavours", or rather more correctly, of definite mass eigenstates, solely constrained by the requirements of Lorentz invariance and electric charge conservation. For instance, even though this might be realized only in peculiar classes of models beyond the SM, allowance is made for the possibility that both the neutrino fields and their charge conjugates couple in the effective Lagrangian density. Furthermore, the analysis is developed separately whether for Dirac or Majorana neutrinos, with the hope to identify circumstances under which scattering experiments involving neutrinos could help discriminate between these two cases through different angular correlations for differential cross sections, given the high rates to be expected at neutrino factories. As is well known, the "practical Dirac-Majorana confusion theorem" states[8] that within the SM, namely in the limit of massless neutrinos as well as $(V-A)$ interactions only, these two possibilities are physically totally equivalent, and hence cannot be distinguished. On the other hand, relaxing the purely $(V-A)$ structure of the electroweak interaction by including at least another interaction whose chirality structure is different, should suffice to evade this conclusion, even in the limit of massless neutrinos.

The general classes of processes comprise neutrino pair annihilation into charged leptons,[1] the inverse process of neutrino pair production through lepton annihilation, and finally neutrino-lepton scattering. These processes will also be considered whether either one or both pairs of neutrino and lepton flavours, (a, b) and (i, j) respectively, are identical or not. The sole implicit assumption is that the energy available to the reaction is both sufficiently large in order to justify ignoring neutrino and lepton masses, and sufficiently small in order to justify the four-fermion parametrization of the boson exchanges responsible for the interactions. Hence, calculations are performed in the limit of zero mass for all external neutrino and lepton mass eigenstates. Nonetheless, effects that distinguish Majorana from Dirac neutrinos survive in this limit. Note that this massless approximation also justifies our

[1] Henceforth, the charged fermions are referred to as leptons, even though exactly the same analysis and results apply to quarks, with due account then for the quark colour degree of freedom and the quark structure of the hadrons involved. Also, charged leptons will simply be called leptons, for short.

abuse of language in referring to neutrino mass eigenstates as flavour eigenstates.

Sect.2 provides a list of useful relations for Dirac and Majorana spinors. Sect.3 discusses the general four-fermion effective Lagrangian used in our analysis. Sects.4 to 6 then list the results for the three classes of processes mentioned above. Some comments and concluding remarks are made in Sect.7.

2 A Compendium of Properties

2.1 Dirac, Weyl and Majorana spinors

This section present facts relevant to Dirac, Weyl and Majorana spinors. Since all processes are considered in the massless limit, the representation of the Clifford-Dirac algebra $\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}=2\,\mathrm{diag}\,(+---)$ used throughout is the chiral one ($\mu=0,1,2,3;\ i=1,2,3$),

$$\gamma^0=\begin{pmatrix}0 & -\mathbb{1}\\ -\mathbb{1} & 0\end{pmatrix}\ ,\ \gamma^i=\begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}\ ,\ \gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3=\begin{pmatrix}\mathbb{1} & 0\\ 0 & -\mathbb{1}\end{pmatrix}\ , \tag{1}$$

σ^i being of course the Pauli matrices. The chiral projectors P_η ($\eta=\pm$) are given by

$$P_\eta=\frac{1}{2}\left[1+\eta\gamma_5\right]\quad,\quad P_\eta^2=P_\eta\quad,\quad P_\eta P_{-\eta}=0\quad,\quad \eta=+,-\ . \tag{2}$$

By definition, the charge conjugation matrix C is such that

$$\begin{array}{rclcrcl}C^{-1}\mathbb{1}C & = & \mathbb{1}^{\mathrm{T}} & , & C^{-1}\gamma_5 C & = & \gamma_5^{\mathrm{T}}\ ,\\ C^{-1}\gamma^\mu C & = & -\gamma^{\mu\mathrm{T}} & , & C^{-1}\left(\gamma^\mu\gamma_5\right)C & = & \left(\gamma^\mu\gamma_5\right)^{\mathrm{T}}\ ,\\ C^{-1}\sigma_{\mu\nu}C & = & -\sigma_{\mu\nu}^{\mathrm{T}} & , & C^{-1}\left(\sigma_{\mu\nu}\gamma_5\right)C & = & -\left(\sigma_{\mu\nu}\gamma_5\right)^{\mathrm{T}}\ ,\end{array} \tag{3}$$

with $C^{\mathrm{T}}=C^\dagger=-C$ and $CC^\dagger=\mathbb{1}=C^\dagger C$, and is realized in the chiral representation by $C=\mathrm{diag}(-i\sigma^2,\ i\sigma^2)$.

Given a four component Dirac spinor ψ, our definition of the associated charge conjugate spinor is such that

$$\psi_c=\psi^c=\lambda\, C\overline{\psi}^{\mathrm{T}}\ , \tag{4}$$

where λ is some arbitrary unit phase factor, whose value may depend on the spinor field.

Solutions to the free massless Dirac equation may be expanded as follows in the helicity basis in the case of a Dirac spinor $\psi_D(x)$,

$$\psi_D(x)=\int_{(\infty)}\frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|}\sum_{\eta=\pm}\left[e^{-ik\cdot x}u(\vec{k},\eta)b(\vec{k},\eta)+e^{ik\cdot x}v(\vec{k},\eta)d^\dagger(\vec{k},\eta)\right]\ . \tag{5}$$

Here, the fermionic creation and annihilation operators have the Lorentz covariant normalization

$$\left\{b(\vec{k},\eta),b^\dagger(\vec{k}',\eta')\right\}=(2\pi)^3 2|\vec{k}|\delta_{\eta,\eta'}\delta^{(3)}(\vec{k}-\vec{k}')=\left\{d(\vec{k},\eta),d^\dagger(\vec{k}',\eta')\right\}\ , \tag{6}$$

while the plane wave spinors $u(\vec{k},\eta)$ and $v(\vec{k},\eta)$ are given by,

$$u(\vec{k},+)=v(\vec{k},-)=\sqrt{2|\vec{k}|}\begin{pmatrix}\chi_+(\hat{k})\\ 0\end{pmatrix}\ ,\ u(\vec{k},-)=v(\vec{k},+)=\sqrt{2|\vec{k}|}\begin{pmatrix}0\\ \chi_-(\hat{k})\end{pmatrix}\ , \tag{7}$$

with the Pauli bi-spinors

$$\chi_+(\hat{k}) = \begin{pmatrix} e^{-i\varphi/2}\cos\theta/2 \\ e^{i\varphi/2}\sin\theta/2 \end{pmatrix} \quad , \quad \chi_-(\hat{k}) = \begin{pmatrix} -e^{-i\varphi/2}\sin\theta/2 \\ e^{i\varphi/2}\cos\theta/2 \end{pmatrix} \ , \tag{8}$$

such that $\hat{k}\cdot\vec{\sigma}\,\chi_\eta(\hat{k}) = \eta\,\chi_\eta(\hat{k})$ and $\chi_\eta(\hat{k})\chi_\eta^\dagger(\hat{k}) = (\mathbb{1}+\eta\hat{k}\cdot\vec{\sigma})/2$, φ and θ being the spherical angles for the unit vector $\hat{k} = \vec{k}/|\vec{k}|$ with respect to the axes $i = 1,2,3$, namely $\hat{k} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$.

The value of $\eta = \pm$ coincides with the helicity of the associated massless one-particle states, as well as the chirality of the associated quantum field. Left- or right-handed four component Weyl spinors, with $\eta = -$ and $\eta = +$ respectively, read

$$\psi_\eta(x) = \int_{(\infty)} \frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|}\left[e^{-ik\cdot x}u(\vec{k},\eta)b(\vec{k},\eta) + e^{ik\cdot x}v(\vec{k},-\eta)d^\dagger(\vec{k},-\eta)\right] \ , \tag{9}$$

as implied by the identification

$$\psi_\eta(x) = P_\eta\,\psi_D(x) \ . \tag{10}$$

Hence, $b^\dagger(\vec{k},\eta)$ and $d^\dagger(\vec{k},\eta)$ are the creation operators of a particle and of an antiparticule, respectively, each of helicity η and momentum $\vec{k}$.

This identification may also be established from the chiral properties of the plane wave spinors,

$$\begin{array}{rclcrcl} P_\eta\,u(\vec{k},\eta) &=& u(\vec{k},\eta) & , & P_\eta\,u(\vec{k},-\eta) &=& 0 \ , \\ P_\eta\,v(\vec{k},\eta) &=& 0 & , & P_\eta\,v(\vec{k},-\eta) &=& v(\vec{k},-\eta) \ , \\ \overline{u}(\vec{k},\eta)\,P_\eta &=& 0 & , & \overline{u}(\vec{k},-\eta)\,P_\eta &=& \overline{u}(\vec{k},-\eta) \ , \\ \overline{v}(\vec{k},\eta)\,P_\eta &=& \overline{v}(\vec{k},\eta) & , & \overline{v}(\vec{k},-\eta)\,P_\eta &=& 0 \ , \end{array} \tag{11}$$

as well as

$$u(\vec{k},\eta)\overline{u}(\vec{k},\eta) = \frac{\mathbb{1}+\eta\gamma_5}{2}\not{k} \quad , \quad v(\vec{k},\eta)\overline{v}(\vec{k},\eta) = \frac{\mathbb{1}-\eta\gamma_5}{2}\not{k} \ . \tag{12}$$

Their properties under charge conjugation are such that $C\overline{u}^{\rm T}(\vec{k},\eta) = v(\vec{k},\eta)$, $C\overline{v}^{\rm T}(\vec{k},\eta) = u(\vec{k},\eta)$, $\overline{v}(\vec{k},\eta) = u^{\rm T}(\vec{k},\eta)C$ and $\overline{u}(\vec{k},\eta) = v^{\rm T}(\vec{k},\eta)C$, these relations being specific to the helicity basis. Charge conjugates of spinors are then given by, say for a Dirac spinor $\psi_D(x)$,

$$\psi^c_D(x) = \int_{(\infty)} \frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|}\sum_{\eta=\pm}\left[e^{-ik\cdot x}\lambda u(\vec{k},\eta)d(\vec{k},\eta) + e^{ik\cdot x}\lambda v(\vec{k},\eta)b^\dagger(\vec{k},\eta)\right] \ . \tag{13}$$

As opposed to a Dirac spinor comprised of two independent Weyl spinors of opposite chiralities, namely one of each of the two fundamental representations of the (covering group of the) Lorentz group, $\psi_D(x) = \psi_+(x) + \psi_-(x)$, a Majorana spinor $\psi_M(x)$ is a four component spinor, thus also covariant under Lorentz transformations, but constructed from a single Weyl spinor, say of left-handed chirality[2] $\eta = -$, and which is invariant under charge conjugation[3]

$$\psi_M(x) = \psi_-(x) + \psi^c_-(x) \quad , \quad \psi^c_M(x) = \lambda_M C\overline{\psi}^{\rm T} = \psi_M(x) \ , \tag{14}$$

[2]Since charge conjugation exchanges left- and right-handed chiralities, the chirality of the basic Weyl spinor used in this construction is irrelevant to the definition of a Majorana spinor.

[3]A similar definition starting from a Dirac rather than a Weyl spinor might be contemplated, leading then to two independent Majorana spinors, each of which is obtained in the manner just described from a single distinct Weyl spinor, namely $\psi^{(1)}_M = (\psi_D + \psi^c_D)/\sqrt{2}$ and $\psi^{(2)}_M = -i(\psi_D - \psi^c_D)/\sqrt{2}$, in complete analogy with the real and imaginary parts of a single complex scalar field as well as the physical interpretation of the associated quanta as being particles which are or not their own antiparticles. Specifically, we have $\psi^{(1)}_M = \psi^{(1)}_- + \psi^{(1)^c}_-$ and $\psi^{(2)}_M = \psi^{(2)}_- + \psi^{(2)^c}_-$ with $\psi^{(1)}_- = (\psi_- + \psi^c_+)/\sqrt{2}$, $\psi^{(2)}_- = -i(\psi_- - \psi^c_+)/\sqrt{2}$, where $\psi_D = \psi_- + \psi_+$. Setting either ψ_- or ψ_+ to zero, the Weyl spinors $\psi^{(1)}_-$, $\psi^{(2)}_-$ hence also the Majorana ones $\psi^{(1)}_M$, $\psi^{(2)}_M$ are then no longer independent, leading back to the construction described in the body of the text.

where the possible spinor dependency of the phase factor λ_M is now emphasized. Consequently, the mode expansion of a Majorana spinor in the helicity basis is,

$$\psi_M(x) = \int_{(\infty)} \frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|} \sum_{\eta=\pm} \left[e^{-ik\cdot x} u(\vec{k},\eta) a(\vec{k},\eta) + e^{ik\cdot x} \lambda_M v(\vec{k},\eta) a^\dagger(\vec{k},\eta) \right] \ , \tag{15}$$

where the annihilation and creation operators $a(\vec{k},\eta)$ and $a^\dagger(\vec{k},\eta)$ obey the fermionic algebra

$$\left\{ a(\vec{k},\eta), a^\dagger(\vec{k}',\eta') \right\} = (2\pi)^3 2|\vec{k}| \delta_{\eta,\eta'} \delta^{(3)}(\vec{k}-\vec{k}') \ . \tag{16}$$

In terms of the quanta of the basic Weyl spinor used in the construction, we have the following correspondence (the complex conjugate of a complex number z is denoted z^* throughout),

$$\begin{array}{lclclcl} a(\vec{k},-) & : & b(\vec{k},-) & ; & a^\dagger(\vec{k},-) & : & b^\dagger(\vec{k},-) \ , \\ a(\vec{k},+) & : & \lambda_M d(\vec{k},+) & ; & a^\dagger(\vec{k},+) & : & \lambda_M^* d^\dagger(\vec{k},+) \ , \end{array} \tag{17}$$

showing that $a^\dagger(\vec{k},\eta)$ is the creation operator of a particle of momentum $\vec{k}$ and helicity η which is also its own antiparticle. The charge conjugation phase factor λ_M is seen to corresponds to the so-callsed "creation phase factor".[9]

2.2 Differential cross sections

All $2 \to 2$ processes to be discussed are considered in their center-of-mass (CM) frame, with a kinematics of the form

$$p_1 + p_2 \to q_1 + q_2 \ , \tag{18}$$

the quantities $p_{1,2}$, $q_{1,2}$ standing for the four-momenta of the respective in-coming and out-going massless particles. Given rotational invariance, and the fact that all particles are of spin 1/2 and of zero mass, hence of definite helicity, the sole angle of relevance is the CM scattering angle θ between, say, the momenta $\vec{p}_1$ and $\vec{q}_1$. For all the reactions listed hereafter, the same order is used for the pairs (p_1,p_2) and (q_1,q_2) of the initial and final particles involved, hence leading always to the same interpretation for this angle θ as being the scattering angle between the first particle in each of these two pairs of in-coming and out-going states.

For external particles of definite helicity, the differential CM cross section of all such processes is given by

$$\frac{d\sigma}{d\Omega_{\hat{q}_1}} = \frac{1}{S_f}\frac{1}{64\pi^2 s}|\mathcal{M}|^2 \quad , \quad \frac{d\sigma}{d\cos\theta} = \frac{1}{S_f}\frac{1}{32\pi\, s}|\mathcal{M}|^2 \ . \tag{19}$$

Here, $\sqrt{s}$ is the reaction total invariant energy, with $s = (p_1+p_2)^2 = (q_1+q_2)^2$, $d\Omega_{\hat{q}_1}$ is the solid angle associated to the outgoing particle of normalized momentum $\hat{q}_1 = \vec{q}_1/|\vec{q}_1|$, $S_f = 2$ or $S_f = 1$ depending on whether the two particles—including their helicity—in the final state are identical or not, respectively, and $\mathcal{M}$ is Feynman's scattering matrix element. Our results are listed in terms of the relevant amplitudes $\mathcal{M}$.

3 The Four-Fermion Effective Lagrangian

Given our assumption concerning the energy and mass scales involved, an effective four-fermion parametrization is warranted, constrained by the sole requirements of Lorentz invariance and electric charge conservation. Since fermion number is not necessarily conserved, one may equally well couple the neutrino fields and their charge conjugates to the charged fermionic fields. For the latter, Dirac fields represent the ordinary charged leptons (or quarks) rather than their antiparticules. It is relative to this choice that the neutrino fields and their charge conjugates are thus specified.

We shall consider all processes involving neutrinos or their antineutrinos of definite flavours a and b, as well as leptons or their antileptons of flavours i and j, all denoted as ν_a, ν_b, ℓ_i^- and ℓ_j^-, respectively. Hence, the total four-fermion effective Lagrangian in the case of Dirac neutrinos is (through a Fierz transformation, its expression may be brought to the charge-retention form),

$$\mathcal{L}_{\rm eff} = 4\frac{g^2}{8M^2}\left[\mathcal{L}_D + \mathcal{L}_D^\dagger\right] \quad , \quad \mathcal{L}_D = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 \ , \tag{20}$$

each separate contribution being given by

$$\begin{aligned}\mathcal{L}_1 = S_1^{\eta_a,\eta_b}\overline{\nu_a}P_{-\eta_a}\ell_i\,\overline{\ell_j}P_{\eta_b}\nu_b \quad &+ V_1^{\eta_a,\eta_b}\overline{\nu_a}\gamma^\mu P_{\eta_a}\ell_i\,\overline{\ell_j}\gamma_\mu P_{\eta_b}\nu_b \\ &+ \tfrac{1}{2}T_1^{\eta_a,\eta_b}\overline{\nu_a}\sigma^{\mu\nu}P_{-\eta_a}\ell_i\,\overline{\ell_j}\sigma_{\mu\nu}P_{\eta_b}\nu_b \ ,\end{aligned} \tag{21}$$

$$\begin{aligned}\mathcal{L}_2 = S_2^{\eta_a,\eta_b}\overline{\nu_a^c}P_{\eta_a}\ell_i\,\overline{\ell_j}P_{\eta_b}\nu_b \quad &+ V_2^{\eta_a,\eta_b}\overline{\nu_a^c}\gamma^\mu P_{-\eta_a}\ell_i\,\overline{\ell_j}\gamma_\mu P_{\eta_b}\nu_b \\ &+ \tfrac{1}{2}T_2^{\eta_a,\eta_b}\overline{\nu_a^c}\sigma^{\mu\nu}P_{\eta_a}\ell_i\,\overline{\ell_j}\sigma_{\mu\nu}P_{\eta_b}\nu_b \ ,\end{aligned} \tag{22}$$

$$\begin{aligned}\mathcal{L}_3 = S_3^{\eta_a,\eta_b}\overline{\nu_a}P_{-\eta_a}\ell_i\,\overline{\ell_j}P_{-\eta_b}\nu_b^c \quad &+ V_3^{\eta_a,\eta_b}\overline{\nu_a}\gamma^\mu P_{\eta_a}\ell_i\,\overline{\ell_j}\gamma_\mu P_{-\eta_b}\nu_b^c \\ &+ \tfrac{1}{2}T_3^{\eta_a,\eta_b}\overline{\nu_a}\sigma^{\mu\nu}P_{-\eta_a}\ell_i\,\overline{\ell_j}\sigma_{\mu\nu}P_{-\eta_b}\nu_b^c \ ,\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{L}_4 = S_4^{\eta_a,\eta_b}\overline{\nu_a^c}P_{\eta_a}\ell_i\,\overline{\ell_j}P_{-\eta_b}\nu_b^c \quad &+ V_4^{\eta_a,\eta_b}\overline{\nu_a^c}\gamma^\mu P_{-\eta_a}\ell_i\,\overline{\ell_j}\gamma_\mu P_{-\eta_b}\nu_b^c \\ &+ \tfrac{1}{2}T_4^{\eta_a,\eta_b}\overline{\nu_a^c}\sigma^{\mu\nu}P_{\eta_a}\ell_i\,\overline{\ell_j}\sigma_{\mu\nu}P_{-\eta_b}\nu_b^c \ ,\end{aligned} \tag{24}$$

an implicit summation over the chiralities η_a and η_b being understood of course. It is important to keep in mind that no summation over the flavour indices a and b, nor i and j is implied; all four of these values are fixed from the outset, keeping open still the possibility that a and b might be equal or not, and likewise for i and j.

The overall normalization factor $4g^2/8M^2$ involves a dimensionless coupling constant g as well as a mass scale M, while the factor 4 cancels the two factors 1/2 present in the chiral projection operators $P_{\pm\eta_a}$ and $P_{\pm\eta_b}$. The rationale for this choice of normalization is that in the limit of the SM, g is then the $\mathrm{SU}(2)_L$ gauge coupling constant g_L and M the $W^\pm$ mass M_W, with the tree-level relation to Fermi's constant, $G_F/\sqrt{2} = g_L^2/(8M_W^2)$.

A complex value for either of the coupling coefficients $\{S,V,T\}_{1,2,3,4}^{\eta_a,\eta_b}$ leads to CP violation. The indices η_a and η_b correspond to the neutrino helicities η_a or η_b, while the lepton helicities are then identical or opposity depending on the chiral structure of the coupling operator. The tensor couplings $T_1^{\eta_a,\eta_b}$ and $T_4^{\eta_a,\eta_b}$ contribute only if $\eta_a = -\eta_b$, while the couplings $T_2^{\eta_a,\eta_b}$ and $T_3^{\eta_a,\eta_b}$ contribute only if $\eta_a = \eta_b$.

For Majorana neutrinos, the parametrization is

$$\mathcal{L}_{\rm eff} = 4\frac{g^2}{8M^2}\left[\mathcal{L}_M + \mathcal{L}_M^\dagger\right] \ , \tag{25}$$

where

$$\begin{aligned}\mathcal{L}_M = S^{\eta_a,\eta_b}\overline{\nu_a}P_{-\eta_a}\ell_i\,\overline{\ell_j}P_{\eta_b}\nu_b \quad &+ V^{\eta_a,\eta_b}\overline{\nu_a}\gamma^\mu P_{\eta_a}\ell_i\,\overline{\ell_j}\gamma_\mu P_{\eta_b}\nu_b \\ &+ \tfrac{1}{2}T^{\eta_a,\eta_b}\overline{\nu_a}\sigma^{\mu\nu}P_{-\eta_a}\ell_i\,\overline{\ell_j}\sigma_{\mu\nu}P_{\eta_b}\nu_b \ .\end{aligned} \tag{26}$$

Compared to the definitions above, and given the property $\psi_M^c = \psi_M$, the correspondence between the effective coupling coefficients in the Majorana and the Dirac cases is obvious. Note that in the Dirac case, the total neutrino number is conserved only for couplings of type 1 and 4, $\{S,V,T\}_{1,4}^{\eta_a,\eta_b}$, whereas the couplings of type 2 and 3, $\{S,V,T\}_{2,3}^{\eta_a,\eta_b}$, violate that quantum number by two units.

In the electroweak Standard Model, besides the normalization factor $4g^2/(8M^2) = 4g_L^2/(8M_W^2)$, in order to identify the nonvanishing couplings, different situations must be distinguished depending on whether only $W^\pm$ or only Z_0 exchanges are involved, or both.

Purely $W^\pm$ exchange processes arise when $a = i$, $b = j$, $a \neq b$ and $i \neq j$, in which case the only nonvanishing effectif coupling is the pure $(V - A)$ one, $V_1^{-,-} = -1$. For purely Z_0 neutral current processes which arise when $(a = b) \neq (i = j)$, the only nonvanishing couplings are $S_1^{-,-} = \sin^2\theta_W$ and $V_1^{-,-} = \frac{1}{4}\left(1 - 2\sin^2\theta_W\right)$, θ_W being the electroweak gauge mixing angle. Finally, charged as well as neutral exchanges both contribute only when all four fermion flavours are identical, $(a = b = i = j)$, leading to the only nonvanishing couplings $S_1^{-,-} = \sin^2\theta_W$ and $V_1^{-,-} = \frac{1}{4}\left(-1 - 2\sin^2\theta_W\right)$. In the last two situations, $\mathcal{L}_D$ and $\mathcal{L}_D^\dagger$, or $\mathcal{L}_M$ and $\mathcal{L}_M^\dagger$, are identical. Any extra coupling beyond these ones thus corresponds to some new physics beyond the SM.

The remainder of the calculation proceeds straightforwardly. Given any choice of external states for the in-coming and out-going particles including their helicities, the substitution of the effective Lagrangian operator enables the direct evaluation of the matrix element $\mathcal{M}$ using the Fock algebra of the creation and annihilation operators. Rather than computing $|\mathcal{M}|^2$ through the usual trace techniques, it is far more efficient to substitute for the $u(\vec{k},\eta)$ and $v(\vec{k},\eta)$ spinors. One then readily obtains the value for $\mathcal{M}$ as a function of θ.

4 Neutrino Pair Annihilation

In the Dirac case, neutrino pair annihilations are labelled as,

$$\begin{array}{llll} \multicolumn{4}{c}{\underline{(ab)(ij)\ \text{Dirac neutrino annihilations}}} \\ \text{ab1: } \nu_a + \nu_b \to \ell_i^- + \ell_j^+ & , & \text{ab2: } \nu_a + \nu_b \to \ell_i^+ + \ell_j^- & , \\ \text{ab3: } \nu_a + \bar\nu_b \to \ell_i^- + \ell_j^+ & , & \text{ab4: } \nu_a + \bar\nu_b \to \ell_i^+ + \ell_j^- & , \\ \text{ab5: } \bar\nu_a + \nu_b \to \ell_i^- + \ell_j^+ & , & \text{ab6: } \bar\nu_a + \nu_b \to \ell_i^+ + \ell_j^- & , \\ \text{ab7: } \bar\nu_a + \bar\nu_b \to \ell_i^- + \ell_j^+ & , & \text{ab8: } \bar\nu_a + \bar\nu_b \to \ell_i^+ + \ell_j^- & , \end{array}$$

while in the Majorana case,

$$\begin{array}{c} \underline{(ab)(ij)\ \text{Majorana neutrino annihilations}} \\ \text{Mab1: } \nu_a + \nu_b \to \ell_i^- + \ell_j^+ \quad , \quad \text{Mab2: } \nu_a + \nu_b \to \ell_i^+ + \ell_j^- \ . \end{array}$$

Due to common angular-momentum selection rules, the matrix element $\mathcal{M}$ for all these ten processes is

$$\begin{array}{ll} \mathcal{M}_{(ab)(ij)} = & \\ = -4s\left(\frac{g^2}{8M^2}\right) & N_1\,\delta_{ij}\left\{\delta_{ab}\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}\left[A_{11}\sin^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{11}(1+\cos^2\theta/2)\right]\right. \\ & +\delta_{ab}\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}C_{11}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\cos^2\theta/2\right] \\ & +\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}\eta_a\eta_b D_1\left[A_{12}\cos^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{12}(1+\sin^2\theta/2)\right] \\ & \left.+\ \delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}\eta_a\eta_b D_1 C_{12}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2\right]\right\} \\ -4s\left(\frac{g^2}{8M^2}\right) & N_2\left\{\delta_{ab}\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}\left[A_{21}\cos^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{21}(1+\sin^2\theta/2)\right]\right. \\ & +\delta_{ab}\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}C_{21}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2\right] \\ & +\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}\eta_a\eta_b D_2\left[A_{22}\sin^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{22}(1+\cos^2\theta/2)\right] \\ & \left.+\ \delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}\eta_a\eta_b D_2 C_{22}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\cos^2\theta/2\right]\right\}\ , \end{array} \tag{27}$$

where θ is the scattering angle between the neutrino of flavour a and the charged lepton of flavour i. The particle helicities are η_a, η_b, η_i and η_j, respectively. Tables 1 and 2 list the values for the constant phase factors $N_{1,2}$ and $D_{1,2}$ and the subsets of the scalar, tensor and vector effective couplings constants, in that order, defining the quantities $A_{11,12,21,22}$, $B_{11,12,21,22}$ and $C_{11,12,21,22}$, whether in the case of Dirac or Majorana neutrinos.

5 Neutrino Pair Production

For the sake of completeness, neutrino pair production has also been considered. In the Dirac case, the following list applies,

$$\begin{array}{ll} \multicolumn{2}{c}{\underline{(ij)(ab)\ \text{Dirac processes}}} \\ \text{ij1: } \ell_i^- + \ell_j^+ \to \nu_a + \nu_b \quad , & \text{ij2: } \ell_i^+ + \ell_j^- \to \nu_a + \nu_b \quad , \\ \text{ij3: } \ell_i^- + \ell_j^+ \to \nu_a + \bar{\nu}_b \quad , & \text{ij4: } \ell_i^+ + \ell_j^- \to \nu_a + \bar{\nu}_b \quad , \\ \text{ij5: } \ell_i^- + \ell_j^+ \to \bar{\nu}_a + \nu_b \quad , & \text{ij6: } \ell_i^+ + \ell_j^- \to \bar{\nu}_a + \nu_b \quad , \\ \text{ij7: } \ell_i^- + \ell_j^+ \to \bar{\nu}_a + \bar{\nu}_b \quad , & \text{ij8: } \ell_i^+ + \ell_j^- \to \bar{\nu}_a + \bar{\nu}_b \quad , \end{array}$$

while in the Majorana case

$$\begin{array}{c} \underline{(ij)(ab)\ \text{Majorana processes}} \\ \text{Mij1: } \ell_i^- + \ell_j^+ \to \nu_a + \nu_b \quad , \quad \text{Mij2: } \ell_i^+ + \ell_j^- \to \nu_a + \nu_b \ . \end{array}$$

For all these ten processes, the amplitude $\mathcal{M}$ is of the form

$$\begin{array}{ll} \mathcal{M}_{(ij)(ab)} = & \\ = 4s\left(\frac{g^2}{8M^2}\right) & N_1\,\delta_{ij}\Big\{\delta_{ab}\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}\left[A_{11}\sin^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{11}(1+\cos^2\theta/2)\right] \\ & -\delta_{ab}\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}C_{11}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\cos^2\theta/2\right] \\ & +\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}D_1\left[A_{12}\cos^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{12}(1+\sin^2\theta/2)\right] \\ & -\ \delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}D_1C_{12}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2\right]\Big\} \\ +4s\left(\frac{g^2}{8M^2}\right) & N_2\,\Big\{\delta_{ab}\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}\left[A_{21}\cos^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{21}(1+\sin^2\theta/2)\right] \\ & -\delta_{ab}\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}C_{21}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\sin^2\theta/2\right] \\ & +\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}D_2\left[A_{22}\sin^2\theta/2 + 2\delta_{\eta_a,\eta_b}B_{22}(1+\cos^2\theta/2)\right] \\ & -\ \delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}D_2C_{22}\left[(1+\eta_a\eta_b)-(1-\eta_a\eta_b)\cos^2\theta/2\right]\Big\}\ , \end{array} \tag{28}$$

θ being the angle between the lepton of flavour i and the neutrino of flavour a. The different factors and coefficients appearing in this expression are detailed in Tables 3 and 4, whether in the case of Dirac or Majorana neutrinos.

6 Neutrino Scattering

In the case of neutrino scattering onto a charged lepton, we list the results only for the classes of processes $(ai)(bj)$ and $(aj)(bi)$, since the other two classes $(bi)(aj)$ and $(bj)(ai)$ may be obtained by appropriate permutations.[6, 7]

6.1 $(ai)(bj)$ neutrino scattering processes

In the case of Dirac neutrinos, the list of processes is

$$\underline{(ai)(bj)\ \text{Dirac processes}}$$

$$\begin{array}{llll}
\text{ai1: } \nu_a + \ell_i^- \to \nu_b + \ell_j^- & , & \text{ai2: } \nu_a + \ell_i^+ \to \nu_b + \ell_j^+ & , \\
\text{ai3: } \nu_a + \ell_i^- \to \bar{\nu}_b + \ell_j^- & , & \text{ai4: } \nu_a + \ell_i^+ \to \bar{\nu}_b + \ell_j^+ & , \\
\text{ai5: } \bar{\nu}_a + \ell_i^- \to \nu_b + \ell_j^- & , & \text{ai6: } \bar{\nu}_a + \ell_i^+ \to \nu_b + \ell_j^+ & , \\
\text{ai7: } \bar{\nu}_a + \ell_i^- \to \bar{\nu}_b + \ell_j^- & , & \text{ai8: } \bar{\nu}_a + \ell_i^+ \to \bar{\nu}_b + \ell_j^+ & ,
\end{array}$$

while in the Majorana case

$$\underline{(ai)(bj)\ \text{Majorana processes}}$$

$$\text{Mai1: } \nu_a + \ell_i^- \to \nu_b + \ell_j^- \quad , \quad \text{Mai2: } \nu_a + \ell_i^+ \to \nu_b + \ell_j^+ \ .$$

The general amplitude $\mathcal{M}$ then reads in all ten cases

$$\begin{aligned}
&\mathcal{M}_{(ai)(bj)} = \\
&= 4s\left(\frac{g^2}{8M^2}\right) \quad N_1\,\delta_{ij}\Big\{\delta_{ab}\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}\left[A_{11} - 2\delta_{\eta_a,-\eta_b}B_{11}(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
&\qquad +\delta_{ab}\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}C_{11}\left[1 + \eta_a\eta_b(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
&\qquad +\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}\eta_a\eta_b D_1\left[A_{12}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{12}(1+\sin^2\theta/2)\right] \\
&\qquad +\,\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}\eta_a\eta_b D_1 C_{12}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2)\right]\Big\} \\
&+4s\left(\frac{g^2}{8M^2}\right) \quad N_2\,\Big\{\delta_{ab}\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}\left[A_{21}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{21}(1+\sin^2\theta/2)\right] \\
&\qquad +\delta_{ab}\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}C_{21}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2\right] \\
&\qquad +\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}\eta_a\eta_b D_2\left[A_{22} - 2\delta_{\eta_a,-\eta_b}B_{22}(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
&\qquad +\,\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}\eta_a\eta_b D_2 C_{22}\left[1 + \eta_a\eta_b(\cos^2\theta/2 - \sin^2\theta/2)\right]\Big\}\ ,
\end{aligned} \tag{29}$$

θ being the neutrino scattering angle. The list of factors and coefficients appearing in this expression is detailed in Tables 5 and 6, both in the Dirac and in the Majorana case.

6.2 $(aj)(bi)$ neutrino scattering processes

The list of processes in the Dirac case is

$$\underline{(aj)(bi)\ \text{Dirac processes}}$$

$$\begin{array}{llll}
\text{aj1: } \nu_a + \ell_j^- \to \nu_b + \ell_i^- & , & \text{aj2: } \nu_a + \ell_j^+ \to \nu_b + \ell_i^+ & , \\
\text{aj3: } \nu_a + \ell_j^- \to \bar{\nu}_b + \ell_i^- & , & \text{aj4: } \nu_a + \ell_j^+ \to \bar{\nu}_b + \ell_i^+ & , \\
\text{aj5: } \bar{\nu}_a + \ell_j^- \to \nu_b + \ell_i^- & , & \text{aj6: } \bar{\nu}_a + \ell_j^+ \to \nu_b + \ell_i^+ & , \\
\text{aj7: } \bar{\nu}_a + \ell_j^- \to \bar{\nu}_b + \ell_i^- & , & \text{aj8: } \bar{\nu}_a + \ell_j^+ \to \bar{\nu}_b + \ell_i^+ & ,
\end{array}$$

while in the Majorana case

$$\underline{(aj)(bi)\ \text{Majorana processes}}$$

$$\text{Maj1: } \nu_a + \ell_j^- \to \nu_b + \ell_i^- \quad , \quad \text{Maj2: } \nu_a + \ell_j^+ \to \nu_b + \ell_i^+ \ .$$

The general scattering amplitude $\mathcal{M}$ is of the form

$$
\begin{array}{rl}
\mathcal{M}_{(aj)(bi)} = & \\
= 4s\left(\frac{g^2}{8M^2}\right) & N_1\,\delta_{ij}\Big\{\delta_{ab}\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}\left[A_{11}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{11}(1+\sin^2\theta/2)\right] \\
& +\delta_{ab}\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}C_{11}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2\right] \\
& +\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}\eta_a\eta_b D_1\left[A_{12} - 2\delta_{\eta_a,-\eta_b}B_{12}(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& +\,\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}\eta_a\eta_b D_1 C_{12}\left[1+\eta_a\eta_b(\cos^2\theta/2 - \sin^2\theta/2)\right]\Big\} \\
+4s\left(\frac{g^2}{8M^2}\right) & N_2\,\Big\{\delta_{ab}\delta_{\eta_i}^{\eta_b}\delta_{\eta_j}^{\eta_a}\left[A_{21} - 2\delta_{\eta_a,-\eta_b}B_{21}(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& +\delta_{ab}\delta_{\eta_i}^{-\eta_b}\delta_{\eta_j}^{-\eta_a}C_{21}\left[1+\eta_a\eta_b(\cos^2\theta/2 - \sin^2\theta/2)\right] \\
& +\delta_{\eta_i}^{-\eta_a}\delta_{\eta_j}^{-\eta_b}\eta_a\eta_b D_2\left[A_{22}\cos^2\theta/2 - 2\delta_{\eta_a,-\eta_b}B_{22}(1+\sin^2\theta/2)\right] \\
& +\,\delta_{\eta_i}^{\eta_a}\delta_{\eta_j}^{\eta_b}\eta_a\eta_b D_2 C_{22}\left[(1+\eta_a\eta_b) - (1-\eta_a\eta_b)\sin^2\theta/2)\right]\Big\}\ ,
\end{array}
\tag{30}
$$

the angle θ being that of the scattered neutrino. Tables 7 and 8 list the relevant factors and coefficients both in the Dirac and in the Majorana case.

7 Concluding Remarks

The above general results[6, 7] provide means to assess directly the sensitivity of neutrino beam experiments in the energy range up to a few ten's of GeV's to different fundamental issues of physics in the neutrino sector, whether new interactions beyond the Standard Model, whether the Dirac or Majorana character of neutrinos. By lack of space, only one illustration of the latter instance is presented.

Scalar or tensor couplings being typically less well constrained than vector ones, let us consider an extra scalar interaction, for either of the following two elastic scattering reactions,

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^- \quad , \quad \nu_\mu + \mu^- \rightarrow \nu_\mu + \mu^-\ . \tag{31}$$

These reactions are of the "ai1" type in the $(ai)(bj)$ class, with $a = b \neq i = j$ in the first case, and $a = b = i = j$ in the second. Assuming that beyond the couplings of the SM, $S_1^{+,-}$ is the sole nonvanishing extra interaction, and considering an unpolarized measurement, the sum over all polarization states reads,

$$
\begin{array}{l}
\sum_{\rm pol.}\ |\mathcal{M}|^2 = (4s)^2\left(\frac{g^2}{8M^2}\right)^2 \times \\
\qquad \times\left\{\left[4{\rm Re}\,V_1^{-,-}\right]^2 + \left[{\rm Re}\,S_1^{-,-}\right]^2(1+\cos\theta)^2 + \frac{1}{4}|S_1^{+,-}|^2(1\pm\cos\theta)^2\right\}\ ,
\end{array}
\tag{32}
$$

where in the last term the upper sign corresponds to the Dirac case, and the lower sign to the Majorana case. Hence indeed, any interaction whose chirality structure differs from the SM one leads to processes in which the angular dependency discriminates between Dirac and Majorana neutrinos. Taking as an illustration a value $|S_1^{+,-}| = 0.10$ which is a typical upper-bound on such a coupling in the leptonic $(e\mu)$ sector[4], one finds a 10% sensitivity in the forward-backward asymmetry, certainly a possibility worth to be explored further within the context of realistic foreseen experimental conditions. In other words, neutrino factories may offer an alternative to neutrinoless double β-decay[10] in establishing the Dirac or Majorana character of neutrinos.

The main purpose of this work[6, 7] has been to provide the general results for the Feynman amplitudes for all possible $2 \rightarrow 2$ processes with two neutrinos. On that basis, it should now be possible to develop a detailed and dedicated analysis, extending similar work within restricted classes of effective couplings,[11] of the potential reach of different such reactions towards the above physics issues, inclusive of the possible discrimination between Dirac and Majorana neutrinos, given a specific design both of neutrino beams and their intensities, and of detector set-ups. Besides the great interest to be found in neutrino scattering experiments, the possibilities offered by intersecting neutrino beams should also not be dismissed offhand without first a dedicated assessment as well, the more so since they could possibly run in parasitic mode in conjunction with other experiments given a proper geometry for the neutrino beams.

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Table 1: The constant factors appearing in (27) for the first five $(ab)(ij)$ Dirac neutrino annihilation processes.

	ab1	ab2	ab3	ab4	ab5
N_1	$\eta_a\lambda_a^*$	$\eta_a\lambda_b^*$	η_a	η_a	η_a
A_{11}	$S_2^{\eta_b,\eta_a}$	$S_3^{\eta_b,\eta_a*}$	$S_1^{-\eta_b,\eta_a}$	$S_4^{-\eta_b,\eta_a*}$	$S_4^{\eta_b,-\eta_a}$
B_{11}	$T_2^{\eta_b,\eta_a}$	$T_3^{\eta_b,\eta_a*}$	$T_1^{-\eta_b,\eta_a}$	$T_4^{-\eta_b,\eta_a*}$	$T_4^{\eta_b,-\eta_a}$
C_{11}	$V_2^{\eta_b,\eta_a}$	$V_3^{\eta_b,\eta_a*}$	$V_1^{-\eta_b,\eta_a}$	$V_4^{-\eta_b,\eta_a*}$	$V_4^{\eta_b,-\eta_a}$
D_1	1	1	$\lambda_a^*\lambda_b$	1	1
A_{12}	$S_2^{\eta_a,\eta_b}$	$S_3^{\eta_a,\eta_b*}$	$S_4^{\eta_a,-\eta_b}$	$S_1^{\eta_a,-\eta_b*}$	$S_1^{-\eta_a,\eta_b}$
B_{12}	$T_2^{\eta_a,\eta_b}$	$T_3^{\eta_a,\eta_b*}$	$T_4^{\eta_a,-\eta_b}$	$T_1^{\eta_a,-\eta_b*}$	$T_1^{-\eta_a,\eta_b}$
C_{12}	$V_2^{\eta_a,\eta_b}$	$V_3^{\eta_a,\eta_b*}$	$V_4^{\eta_a,-\eta_b}$	$V_1^{\eta_a,-\eta_b*}$	$V_1^{-\eta_a,\eta_b}$
N_2	$\eta_b\lambda_b^*$	$\eta_b\lambda_a^*$	η_b	η_b	η_b
A_{21}	$S_3^{\eta_b,\eta_a*}$	$S_2^{\eta_b,\eta_a}$	$S_4^{-\eta_b,\eta_a*}$	$S_1^{-\eta_b,\eta_a}$	$S_1^{\eta_b,-\eta_a*}$
B_{21}	$T_3^{\eta_b,\eta_a*}$	$T_2^{\eta_b,\eta_a}$	$T_4^{-\eta_b,\eta_a*}$	$T_1^{-\eta_b,\eta_a}$	$T_1^{\eta_b,-\eta_a*}$
C_{21}	$V_3^{\eta_b,\eta_a*}$	$V_2^{\eta_b,\eta_a}$	$V_4^{-\eta_b,\eta_a*}$	$V_1^{-\eta_b,\eta_a}$	$V_1^{\eta_b,-\eta_a*}$
D_2	1	1	1	$\lambda_a^*\lambda_b$	$\lambda_a\lambda_b^*$
A_{22}	$S_3^{\eta_a,\eta_b*}$	$S_2^{\eta_a,\eta_b}$	$S_1^{\eta_a,-\eta_b*}$	$S_4^{\eta_a,-\eta_b}$	$S_4^{-\eta_a,\eta_b*}$
B_{22}	$T_3^{\eta_a,\eta_b*}$	$T_2^{\eta_a,\eta_b}$	$T_1^{\eta_a,-\eta_b*}$	$T_4^{\eta_a,-\eta_b}$	$T_4^{-\eta_a,\eta_b*}$
C_{22}	$V_3^{\eta_a,\eta_b*}$	$V_2^{\eta_a,\eta_b}$	$V_1^{\eta_a,-\eta_b*}$	$V_4^{\eta_a,-\eta_b}$	$V_4^{-\eta_a,\eta_b*}$

Table 2: The constant factors appearing in (27) for the last three $(ab)(ij)$ Dirac neutrino annihilation processes, and the two Majorana neutrino ones.

	ab6	ab7	ab8	Mab1	Mab2
N_1	η_a	$\eta_a\lambda_b$	$\eta_a\lambda_a$	$\eta_a\lambda_a^*$	$\eta_a\lambda_b^*$
A_{11}	$S_1^{\eta_b,-\eta_a*}$	$S_3^{-\eta_b,-\eta_a}$	$S_2^{-\eta_b,-\eta_a*}$	$S^{-\eta_b,\eta_a}$	$S^{\eta_b,-\eta_a*}$
B_{11}	$T_1^{\eta_b,-\eta_a*}$	$T_3^{-\eta_b,-\eta_a}$	$T_2^{-\eta_b,-\eta_a*}$	$T^{-\eta_b,\eta_a}$	$T^{\eta_b,-\eta_a*}$
C_{11}	$V_1^{\eta_b,-\eta_a*}$	$V_3^{-\eta_b,-\eta_a}$	$V_2^{-\eta_b,-\eta_a*}$	$V^{-\eta_b,\eta_a}$	$V^{\eta_b,-\eta_a*}$
D_1	$\lambda_a\lambda_b^*$	1	1	1	1
A_{12}	$S_4^{-\eta_a,\eta_b*}$	$S_3^{-\eta_a,-\eta_b}$	$S_2^{-\eta_a,-\eta_b*}$	$S^{-\eta_a,\eta_b}$	$S^{\eta_a,-\eta_b*}$
B_{12}	$T_4^{-\eta_a,\eta_b*}$	$T_3^{-\eta_a,-\eta_b}$	$T_2^{-\eta_a,-\eta_b*}$	$T^{-\eta_a,\eta_b}$	$T^{\eta_a,-\eta_b*}$
C_{12}	$V_4^{-\eta_a,\eta_b*}$	$V_3^{-\eta_a,-\eta_b}$	$V_2^{-\eta_a,-\eta_b*}$	$V^{-\eta_a,\eta_b}$	$V^{\eta_a,-\eta_b*}$
N_2	η_b	$\eta_b\lambda_a$	$\eta_b\lambda_b$	$\eta_b\lambda_b^*$	$\eta_b\lambda_a^*$
A_{21}	$S_4^{\eta_b,-\eta_a}$	$S_2^{-\eta_b,-\eta_a*}$	$S_3^{-\eta_b,-\eta_a}$	$S^{\eta_b,-\eta_a*}$	$S^{-\eta_b,\eta_a}$
B_{21}	$T_4^{\eta_b,-\eta_a}$	$T_2^{-\eta_b,-\eta_a*}$	$T_3^{-\eta_b,-\eta_a}$	$T^{\eta_b,-\eta_a*}$	$T^{-\eta_b,\eta_a}$
C_{21}	$V_4^{\eta_b,-\eta_a}$	$V_2^{-\eta_b,-\eta_a*}$	$V_3^{-\eta_b,-\eta_a}$	$V^{\eta_b,-\eta_a*}$	$V^{-\eta_b,\eta_a}$
D_2	1	1	1	1	1
A_{22}	$S_1^{-\eta_a,\eta_b}$	$S_2^{-\eta_a,-\eta_b*}$	$S_3^{-\eta_a,-\eta_b}$	$S^{\eta_a,-\eta_b*}$	$S^{-\eta_a,\eta_b}$
B_{22}	$T_1^{-\eta_a,\eta_b}$	$T_2^{-\eta_a,-\eta_b*}$	$T_3^{-\eta_a,-\eta_b}$	$T^{\eta_a,-\eta_b*}$	$T^{-\eta_a,\eta_b}$
C_{22}	$V_1^{-\eta_a,\eta_b}$	$V_2^{-\eta_a,-\eta_b*}$	$V_3^{-\eta_a,-\eta_b}$	$V^{\eta_a,-\eta_b*}$	$V^{-\eta_a,\eta_b}$

Table 3: The constant factors appearing in (28) for the first five $(ij)(ab)$ Dirac neutrino pair production processes.

	ij1	ij2	ij3	ij4	ij5
N_1	$\eta_a\lambda_a$	$\eta_a\lambda_b$	η_a	η_a	η_a
A_{11}	$S_2^{\eta_b,\eta_a*}$	$S_3^{\eta_b,\eta_a}$	$S_1^{-\eta_b,\eta_a*}$	$S_4^{-\eta_b,\eta_a}$	$S_4^{\eta_b,-\eta_a*}$
B_{11}	$T_2^{\eta_b,\eta_a*}$	$T_3^{\eta_b,\eta_a}$	$T_1^{-\eta_b,\eta_a*}$	$T_4^{-\eta_b,\eta_a}$	$T_4^{\eta_b,-\eta_a*}$
C_{11}	$V_2^{\eta_b,\eta_a*}$	$V_3^{\eta_b,\eta_a}$	$V_1^{-\eta_b,\eta_a*}$	$V_4^{-\eta_b,\eta_a}$	$V_4^{\eta_b,-\eta_a*}$
D_1	1	1	$\lambda_a\lambda_b^*$	1	1
A_{12}	$S_2^{\eta_a,\eta_b*}$	$S_3^{\eta_a,\eta_b}$	$S_4^{\eta_a,-\eta_b*}$	$S_1^{\eta_a,-\eta_b}$	$S_1^{-\eta_a,\eta_b*}$
B_{12}	$T_2^{\eta_a,\eta_b*}$	$T_3^{\eta_a,\eta_b}$	$T_4^{\eta_a,-\eta_b*}$	$T_1^{\eta_a,-\eta_b}$	$T_1^{-\eta_a,\eta_b*}$
C_{12}	$V_2^{\eta_a,\eta_b*}$	$V_3^{\eta_a,\eta_b}$	$V_4^{\eta_a,-\eta_b*}$	$V_1^{\eta_a,-\eta_b}$	$V_1^{-\eta_a,\eta_b*}$
N_2	$\eta_a\lambda_b$	$\eta_a\lambda_a$	η_a	η_a	η_a
A_{21}	$S_3^{\eta_b,\eta_a}$	$S_2^{\eta_b,\eta_a*}$	$S_4^{-\eta_b,\eta_a}$	$S_1^{-\eta_b,\eta_a*}$	$S_1^{\eta_b,-\eta_a}$
B_{21}	$T_3^{\eta_b,\eta_a}$	$T_2^{\eta_b,\eta_a*}$	$T_4^{-\eta_b,\eta_a}$	$T_1^{-\eta_b,\eta_a*}$	$T_1^{\eta_b,-\eta_a}$
C_{21}	$V_3^{\eta_b,\eta_a}$	$V_2^{\eta_b,\eta_a*}$	$V_4^{-\eta_b,\eta_a}$	$V_1^{-\eta_b,\eta_a*}$	$V_1^{\eta_b,-\eta_a}$
D_2	1	1	1	$\lambda_a\lambda_b^*$	$\lambda_a^*\lambda_b$
A_{22}	$S_3^{\eta_a,\eta_b}$	$S_2^{\eta_a,\eta_b*}$	$S_1^{\eta_a,-\eta_b}$	$S_4^{\eta_a,-\eta_b*}$	$S_4^{-\eta_a,\eta_b}$
B_{22}	$T_3^{\eta_a,\eta_b}$	$T_2^{\eta_a,\eta_b*}$	$T_1^{\eta_a,-\eta_b}$	$T_4^{\eta_a,-\eta_b*}$	$T_4^{-\eta_a,\eta_b}$
C_{22}	$V_3^{\eta_a,\eta_b}$	$V_2^{\eta_a,\eta_b*}$	$V_1^{\eta_a,-\eta_b}$	$V_4^{\eta_a,-\eta_b*}$	$V_4^{-\eta_a,\eta_b}$

Table 4: The constant factors appearing in (28) for the last three $(ij)(ab)$ Dirac neutrino pair production processes, and the two Majorana neutrino ones.

	ij6	ij7	ij8	Mij1	Mij2
N_1	η_a	$\eta_a\lambda_b^*$	$\eta_a\lambda_a^*$	$\eta_a\lambda_a$	$\eta_a\lambda_b$
A_{11}	$S_1^{\eta_b,-\eta_a}$	$S_3^{-\eta_b,-\eta_a*}$	$S_2^{-\eta_b,-\eta_a}$	$S^{-\eta_b,\eta_a*}$	$S^{\eta_b,-\eta_a}$
B_{11}	$T_1^{\eta_b,-\eta_a}$	$T_3^{-\eta_b,-\eta_a*}$	$T_2^{-\eta_b,-\eta_a}$	$T^{-\eta_b,\eta_a*}$	$T^{\eta_b,-\eta_a}$
C_{11}	$V_1^{\eta_b,-\eta_a}$	$V_3^{-\eta_b,-\eta_a*}$	$V_2^{-\eta_b,-\eta_a}$	$V^{-\eta_b,\eta_a*}$	$V^{\eta_b,-\eta_a}$
D_1	$\lambda_a^*\lambda_b$	1	1	1	1
A_{12}	$S_4^{-\eta_a,\eta_b}$	$S_3^{-\eta_a,-\eta_b*}$	$S_2^{-\eta_a,-\eta_b}$	$S^{-\eta_a,\eta_b*}$	$S^{\eta_a,-\eta_b}$
B_{12}	$T_4^{-\eta_a,\eta_b}$	$T_3^{-\eta_a,-\eta_b*}$	$T_2^{-\eta_a,-\eta_b}$	$T^{-\eta_a,\eta_b*}$	$T^{\eta_a,-\eta_b}$
C_{12}	$V_4^{-\eta_a,\eta_b}$	$V_3^{-\eta_a,-\eta_b*}$	$V_2^{-\eta_a,-\eta_b}$	$V^{-\eta_a,\eta_b*}$	$V^{\eta_a,-\eta_b}$
N_2	η_a	$\eta_a\lambda_a^*$	$\eta_a\lambda_b^*$	$\eta_a\lambda_b$	$\eta_a\lambda_a$
A_{21}	$S_4^{\eta_b,-\eta_a*}$	$S_2^{-\eta_b,-\eta_a}$	$S_3^{-\eta_b,-\eta_a*}$	$S^{\eta_b,-\eta_a}$	$S^{-\eta_b,\eta_a*}$
B_{21}	$T_4^{\eta_b,-\eta_a*}$	$T_2^{-\eta_b,-\eta_a}$	$T_3^{-\eta_b,-\eta_a*}$	$T^{\eta_b,-\eta_a}$	$T^{-\eta_b,\eta_a*}$
C_{21}	$V_4^{\eta_b,-\eta_a*}$	$V_2^{-\eta_b,-\eta_a}$	$V_3^{-\eta_b,-\eta_a*}$	$V^{\eta_b,-\eta_a}$	$V^{-\eta_b,\eta_a*}$
D_2	1	1	1	1	1
A_{22}	$S_1^{-\eta_a,\eta_b*}$	$S_2^{-\eta_a,-\eta_b}$	$S_3^{-\eta_a,-\eta_b*}$	$S^{\eta_a,-\eta_b}$	$S^{-\eta_a,\eta_b*}$
B_{22}	$T_1^{-\eta_a,\eta_b*}$	$T_2^{-\eta_a,-\eta_b}$	$T_3^{-\eta_a,-\eta_b*}$	$T^{\eta_a,-\eta_b}$	$T^{-\eta_a,\eta_b*}$
C_{22}	$V_1^{-\eta_a,\eta_b*}$	$V_2^{-\eta_a,-\eta_b}$	$V_3^{-\eta_a,-\eta_b*}$	$V^{\eta_a,-\eta_b}$	$V^{-\eta_a,\eta_b*}$

Table 5: The constant factors appearing in (29) for the first five $(ai)(bj)$ Dirac neutrino scattering processes.

	ai1	ai2	ai3	ai4	ai5
N_1	η_a	η_a	$\eta_a\lambda_b^*$	$\eta_a\lambda_a^*$	$\eta_a\lambda_a$
A_{11}	$S_4^{\eta_b,\eta_a*}$	$S_1^{\eta_b,\eta_a}$	$S_3^{-\eta_b,\eta_a*}$	$S_2^{-\eta_b,\eta_a}$	$S_2^{\eta_b,-\eta_a*}$
B_{11}	$T_4^{\eta_b,\eta_a*}$	$T_1^{\eta_b,\eta_a}$	$T_3^{-\eta_b,\eta_a*}$	$T_2^{-\eta_b,\eta_a}$	$T_2^{\eta_b,-\eta_a*}$
C_{11}	$V_4^{\eta_b,\eta_a*}$	$V_1^{\eta_b,\eta_a}$	$V_3^{-\eta_b,\eta_a*}$	$V_2^{-\eta_b,\eta_a}$	$V_2^{\eta_b,-\eta_a*}$
D_1	1	$\lambda_a^*\lambda_b$	1	1	1
A_{12}	$S_1^{\eta_a,\eta_b*}$	$S_4^{\eta_a,\eta_b}$	$S_3^{\eta_a,-\eta_b*}$	$S_2^{\eta_a,-\eta_b}$	$S_2^{-\eta_a,\eta_b*}$
B_{12}	$T_1^{\eta_a,\eta_b*}$	$T_4^{\eta_a,\eta_b}$	$T_3^{\eta_a,-\eta_b*}$	$T_2^{\eta_a,-\eta_b}$	$T_2^{-\eta_a,\eta_b*}$
C_{12}	$V_1^{\eta_a,\eta_b*}$	$V_4^{\eta_a,\eta_b}$	$V_3^{\eta_a,-\eta_b*}$	$V_2^{\eta_a,-\eta_b}$	$V_2^{-\eta_a,\eta_b*}$
N_2	η_b	η_b	$\eta_b\lambda_a^*$	$\eta_b\lambda_b^*$	$\eta_b\lambda_b$
A_{21}	$S_1^{\eta_b,\eta_a}$	$S_4^{\eta_b,\eta_a*}$	$S_2^{-\eta_b,\eta_a}$	$S_3^{-\eta_b,\eta_a*}$	$S_3^{\eta_b,-\eta_a}$
B_{21}	$T_1^{\eta_b,\eta_a}$	$T_4^{\eta_b,\eta_a*}$	$T_2^{-\eta_b,\eta_a}$	$T_3^{-\eta_b,\eta_a*}$	$T_3^{\eta_b,-\eta_a}$
C_{21}	$V_1^{\eta_b,\eta_a}$	$V_4^{\eta_b,\eta_a*}$	$V_2^{-\eta_b,\eta_a}$	$V_3^{-\eta_b,\eta_a*}$	$V_3^{\eta_b,-\eta_a}$
D_2	$\lambda_a^*\lambda_b$	1	1	1	1
A_{22}	$S_4^{\eta_a,\eta_b}$	$S_1^{\eta_a,\eta_b*}$	$S_2^{\eta_a,-\eta_b}$	$S_3^{\eta_a,-\eta_b*}$	$S_3^{-\eta_a,\eta_b}$
B_{22}	$T_4^{\eta_a,\eta_b}$	$T_1^{\eta_a,\eta_b*}$	$T_2^{\eta_a,-\eta_b}$	$T_3^{\eta_a,-\eta_b*}$	$T_3^{-\eta_a,\eta_b}$
C_{22}	$V_4^{\eta_a,\eta_b}$	$V_1^{\eta_a,\eta_b*}$	$V_2^{\eta_a,-\eta_b}$	$V_3^{\eta_a,-\eta_b*}$	$V_3^{-\eta_a,\eta_b}$

Table 6: The constant factors appearing in (29) for the last three $(ai)(bj)$ Dirac neutrino scattering processes, and the two Majorana neutrino ones.

	ai6	ai7	ai8	Mai1	Mai2
N_1	$\eta_a\lambda_b$	η_a	η_a	η_a	η_a
A_{11}	$S_3^{\eta_b,-\eta_a}$	$S_1^{-\eta_b,-\eta_a*}$	$S_4^{-\eta_b,-\eta_a}$	$S^{-\eta_b,-\eta_a*}$	S^{η_b,η_a}
B_{11}	$T_3^{\eta_b,-\eta_a}$	$T_1^{-\eta_b,-\eta_a*}$	$T_4^{-\eta_b,-\eta_a}$	$T^{-\eta_b,-\eta_a*}$	T^{η_b,η_a}
C_{11}	$V_3^{\eta_b,-\eta_a}$	$V_1^{-\eta_b,-\eta_a*}$	$V_4^{-\eta_b,-\eta_a}$	$V^{-\eta_b,-\eta_a*}$	V^{η_b,η_a}
D_1	1	$\lambda_a\lambda_b^*$	1	1	$\lambda_a^*\lambda_b$
A_{12}	$S_3^{-\eta_a,\eta_b}$	$S_4^{-\eta_a,-\eta_b*}$	$S_1^{-\eta_a,-\eta_b}$	S^{η_a,η_b*}	$S^{-\eta_a,-\eta_b}$
B_{12}	$T_3^{-\eta_a,\eta_b}$	$T_4^{-\eta_a,-\eta_b*}$	$T_1^{-\eta_a,-\eta_b}$	T^{η_a,η_b*}	$T^{-\eta_a,-\eta_b}$
C_{12}	$V_3^{-\eta_a,\eta_b}$	$V_4^{-\eta_a,-\eta_b*}$	$V_1^{-\eta_a,-\eta_b}$	V^{η_a,η_b*}	$V^{-\eta_a,-\eta_b}$
N_2	$\eta_b\lambda_a$	η_b	η_b	η_b	η_b
A_{21}	$S_2^{\eta_b,-\eta_a*}$	$S_4^{-\eta_b,-\eta_a}$	$S_1^{-\eta_b,-\eta_a*}$	S^{η_b,η_a}	$S^{-\eta_b,-\eta_a*}$
B_{21}	$T_2^{\eta_b,-\eta_a*}$	$T_4^{-\eta_b,-\eta_a}$	$T_1^{-\eta_b,-\eta_a*}$	T^{η_b,η_a}	$T^{-\eta_b,-\eta_a*}$
C_{21}	$V_2^{\eta_b,-\eta_a*}$	$V_4^{-\eta_b,-\eta_a}$	$V_1^{-\eta_b,-\eta_a*}$	V^{η_b,η_a}	$V^{-\eta_b,-\eta_a*}$
D_2	1	1	$\lambda_a\lambda_b^*$	$\lambda_a^*\lambda_b$	1
A_{22}	$S_2^{-\eta_a,\eta_b*}$	$S_1^{-\eta_a,-\eta_b}$	$S_4^{-\eta_a,-\eta_b*}$	$S^{-\eta_a,-\eta_b}$	S^{η_a,η_b}
B_{22}	$T_2^{-\eta_a,\eta_b*}$	$T_1^{-\eta_a,-\eta_b}$	$T_4^{-\eta_a,-\eta_b*}$	$T^{-\eta_a,-\eta_b}$	T^{η_a,η_b}
C_{22}	$V_2^{-\eta_a,\eta_b*}$	$V_1^{-\eta_a,-\eta_b}$	$V_4^{-\eta_a,-\eta_b*}$	$V^{-\eta_a,-\eta_b}$	V^{η_a,η_b}

Table 7: The constant factors appearing in (30) for the first five $(aj)(bi)$ Dirac neutrino scattering processes.

	aj1	aj2	aj3	aj4	aj5
N_1	η_b	η_b	$\eta_b\lambda_a^*$	$\eta_b\lambda_b^*$	$\eta_b\lambda_b$
A_{11}	$S_1^{\eta_b,\eta_a}$	$S_4^{\eta_b,\eta_a*}$	$S_2^{-\eta_b,\eta_a}$	$S_3^{-\eta_b,\eta_a*}$	$S_3^{\eta_b,-\eta_a}$
B_{11}	$T_1^{\eta_b,\eta_a}$	$T_4^{\eta_b,\eta_a*}$	$T_2^{-\eta_b,\eta_a}$	$T_3^{-\eta_b,\eta_a*}$	$T_3^{\eta_b,-\eta_a}$
C_{11}	$V_1^{\eta_b,\eta_a}$	$V_4^{\eta_b,\eta_a*}$	$V_2^{-\eta_b,\eta_a}$	$V_3^{-\eta_b,\eta_a*}$	$V_3^{\eta_b,-\eta_a}$
D_1	$\lambda_a^*\lambda_b$	1	1	1	1
A_{12}	$S_4^{\eta_a,\eta_b}$	$S_1^{\eta_a,\eta_b*}$	$S_2^{\eta_a,-\eta_b}$	$S_3^{\eta_a,-\eta_b*}$	$S_3^{-\eta_a,\eta_b}$
B_{12}	$T_4^{\eta_a,\eta_b}$	$T_1^{\eta_a,\eta_b*}$	$T_2^{\eta_a,-\eta_b}$	$T_3^{\eta_a,-\eta_b*}$	$T_3^{-\eta_a,\eta_b}$
C_{12}	$V_4^{\eta_a,\eta_b}$	$V_1^{\eta_a,\eta_b*}$	$V_2^{\eta_a,-\eta_b}$	$V_3^{\eta_a,-\eta_b*}$	$V_3^{-\eta_a,\eta_b}$
N_2	η_a	η_a	$\eta_a\lambda_b^*$	$\eta_a\lambda_a^*$	$\eta_a\lambda_a$
A_{21}	$S_4^{\eta_b,\eta_a*}$	$S_1^{\eta_b,\eta_a}$	$S_3^{-\eta_b,\eta_a*}$	$S_2^{-\eta_b,\eta_a}$	$S_2^{\eta_b,-\eta_a*}$
B_{21}	$T_4^{\eta_b,\eta_a*}$	$T_1^{\eta_b,\eta_a}$	$T_3^{-\eta_b,\eta_a*}$	$T_2^{-\eta_b,\eta_a}$	$T_2^{\eta_b,-\eta_a*}$
C_{21}	$V_4^{\eta_b,\eta_a*}$	$V_1^{\eta_b,\eta_a}$	$V_3^{-\eta_b,\eta_a*}$	$V_2^{-\eta_b,\eta_a}$	$V_2^{\eta_b,-\eta_a*}$
D_2	1	$\lambda_a^*\lambda_b$	1	1	1
A_{22}	$S_1^{\eta_a,\eta_b*}$	$S_4^{\eta_a,\eta_b}$	$S_3^{\eta_a,-\eta_b*}$	$S_2^{\eta_a,-\eta_b}$	$S_2^{-\eta_a,\eta_b*}$
B_{22}	$T_1^{\eta_a,\eta_b*}$	$T_4^{\eta_a,\eta_b}$	$T_3^{\eta_a,-\eta_b*}$	$T_2^{\eta_a,-\eta_b}$	$T_2^{-\eta_a,\eta_b*}$
C_{22}	$V_1^{\eta_a,\eta_b*}$	$V_4^{\eta_a,\eta_b}$	$V_3^{\eta_a,-\eta_b*}$	$V_2^{\eta_a,-\eta_b}$	$V_2^{-\eta_a,\eta_b*}$

Table 8: The constant factors appearing in (30) for the last three $(aj)(bi)$ Dirac neutrino scattering processes, and the two Majorana neutrino ones.

	aj6	aj7	aj8	Maj1	Maj2
N_1	$\eta_b\lambda_a$	η_b	η_b	η_b	η_b
A_{11}	$S_2^{\eta_b,-\eta_a*}$	$S_4^{-\eta_b,-\eta_a}$	$S_1^{-\eta_b,-\eta_a*}$	S^{η_b,η_a}	$S^{-\eta_b,-\eta_a*}$
B_{11}	$T_2^{\eta_b,-\eta_a*}$	$T_4^{-\eta_b,-\eta_a}$	$T_1^{-\eta_b,-\eta_a*}$	T^{η_b,η_a}	$T^{-\eta_b,-\eta_a*}$
C_{11}	$V_2^{\eta_b,-\eta_a*}$	$V_4^{-\eta_b,-\eta_a}$	$V_1^{-\eta_b,-\eta_a*}$	V^{η_b,η_a}	$V^{-\eta_b,-\eta_a*}$
D_1	1	1	$\lambda_a\lambda_b^*$	$\lambda_a^*\lambda_b$	1
A_{12}	$S_2^{-\eta_a,\eta_b*}$	$S_1^{-\eta_a,-\eta_b}$	$S_4^{-\eta_a,-\eta_b*}$	$S^{-\eta_a,-\eta_b}$	S^{η_a,η_b*}
B_{12}	$T_2^{-\eta_a,\eta_b*}$	$T_1^{-\eta_a,-\eta_b}$	$T_4^{-\eta_a,-\eta_b*}$	$T^{-\eta_a,-\eta_b}$	T^{η_a,η_b*}
C_{12}	$V_2^{-\eta_a,\eta_b*}$	$V_1^{-\eta_a,-\eta_b}$	$V_4^{-\eta_a,-\eta_b*}$	$V^{-\eta_a,-\eta_b}$	V^{η_a,η_b*}
N_2	$\eta_a\lambda_b$	η_a	η_a	η_a	η_a
A_{21}	$S_3^{\eta_b,-\eta_a}$	$S_1^{-\eta_b,-\eta_a*}$	$S_4^{-\eta_b,-\eta_a}$	$S^{-\eta_b,-\eta_a*}$	S^{η_b,η_a}
B_{21}	$T_3^{\eta_b,-\eta_a}$	$T_1^{-\eta_b,-\eta_a*}$	$T_4^{-\eta_b,-\eta_a}$	$T^{-\eta_b,-\eta_a*}$	T^{η_b,η_a}
C_{21}	$V_3^{\eta_b,-\eta_a}$	$V_1^{-\eta_b,-\eta_a*}$	$V_4^{-\eta_b,-\eta_a}$	$V^{-\eta_b,-\eta_a*}$	V^{η_b,η_a}
D_2	1	$\lambda_a\lambda_b^*$	1	1	$\lambda_a^*\lambda_b$
A_{22}	$S_3^{-\eta_a,\eta_b}$	$S_4^{-\eta_a,-\eta_b*}$	$S_1^{-\eta_a,-\eta_b}$	S^{η_a,η_b*}	$S^{-\eta_a,-\eta_b}$
B_{22}	$T_3^{-\eta_a,\eta_b}$	$T_4^{-\eta_a,-\eta_b*}$	$T_1^{-\eta_a,-\eta_b}$	T^{η_a,η_b*}	$T^{-\eta_a,-\eta_b}$
C_{22}	$V_3^{-\eta_a,\eta_b}$	$V_4^{-\eta_a,-\eta_b*}$	$V_1^{-\eta_a,-\eta_b}$	V^{η_a,η_b*}	$V^{-\eta_a,-\eta_b}$